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A SCHEME OF FRACTIONAL STEPS FOR A NONSTEADY INTERNAL CONJUGATE PROBLEM OF HEAT TRANSFER IN FLOW OF AN INCOMPRESSIBLE LIQUID WITH VARIABLE THERMOPHYSICAL PROPERTIES

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The conjugate problem of heat transfer during the non steady laminar flow of a viscous incompressible liquid at the entrance section of a plane, annular, or cylindrical channel or in a closed region is discussed.

For the calculation of transitional processes in the flow of cryogenic and high-temperature liquids in channels, for the calculation of transitional processes under conditions of free and free–forced convection in channels and closed regions, etc. it is necessary to create methods for the solution of internal conjugate problems of heat exchange allowing for the nonsteadiness and two-dimensionality of the processes of flow and heat transfer and the true temperature dependence of the properties of the liquid and the wall materials. The application of analytical methods for the solutions of conjugate problems in such a formulation is difficult. An economical, convergent, nonlinear, difference scheme which approximates the stated problem is suggested in the present report.

The nonsteady two-dimensional laminar flow of a viscous incompressible liquid in a plane, annular, or cylindrical channel is analyzed. The viscosity, heat capacity, and thermal conductivity of the liquid depend in a known way on the temperature, the density of the walls depends on the coordinates, and the heat capacity and thermal conductivity depend on the coordinates and the temperature. Heat release occurs in the channel walls and in the liquid. The amount of heat released per unit time per unit mass is a known function of the coordinates and time. A mass force, which depends on the coordinates, time, and the temperature acts on the liquid. The temperature distribution over the ends and outer surfaces of the channel walls is known and varies with time. The pressure in the channel varies continuously and at the exit it equals the pressure of the surrounding medium, which depends on time in a known way. At the contact surfaces between the liquid and the walls a coolant is supplied, the rate of inflow of which is known, while the enthalpy depends on the temperature in a known way. It is assumed that before the start of the process a known steady flow of liquid existed in the channel with a known temperature distribution for the liquid and the walls. At the starting time some perturbation of the velocity and temperature is supplied to the entrance, and heat release and the inflow of coolant begin. The nonsteady process which develops is analyzed. The conditions of temperature conjugation are set up at the liquid–wall contact surfaces in the form of boundary conditions of the fourth kind. To set up the boundary conditions at the exit cut of the channel, simplifying assumptions are made. It is assumed that the channel is long enough, and the coolant supply and the heat sources are concentrated in the entrance section, so that the flow becomes one-dimensional near the exit. It is also assumed that the longitudinal heat flux, due to the heat conduction of the liquid, can be estimated and assigned in the form of a known function of time near the exit.

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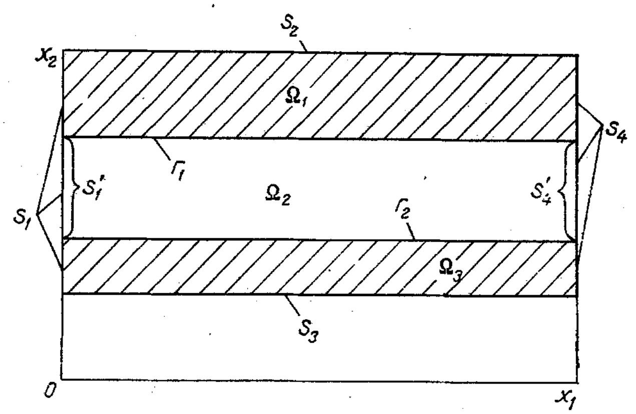

Fig. 1. Diagram of calculated region.

For flow in a closed region the conditions of attachment for the velocity and uniform conditions of conjugation for the temperature are set up at the liquid–wall contact surfaces. Boundary conditions with respect to the temperature are set up at the outer surfaces of the walls. The model of a liquid with a constant density which is used can also be applied to the calculation of flows of dropping liquids, as well as to problems of free and free–forced convection in gases, if one assumes that only the coefficient of volumetric expansion, which enters into the expression for the mass force, depends on the temperature.

In the case of liquid flow in a channel the problem is formulated in the following way. A diagram of the calculated region is shown in Fig. 1, where the designations of its parts and their boundaries are presented. The liquid flow is described by the Navier–Stokes equations (1) and (2) and the heat exchange by the equations of heat conduction and energy (3), which in accordance with [1] have the form

$$\frac{\partial}{\partial x_k}(x_2^\sigma u_k) = 0, \quad X \in \Omega_2; \tag{1}$$

$$x_2^\sigma \left\{ \frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} \right\} = -x_2^\sigma \left(\frac{\partial q}{\partial x_i} - f_i(X, t, \tau) \right) + \frac{\partial}{\partial x_k} \left\{ x_2^\sigma \nu(\tau) \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \right\} - \sigma \delta_i^2 \frac{2\nu(\tau) u_2}{x_2}, \quad i = 1, 2, \; X \in \Omega_2; \tag{2}$$

$$x_2^\sigma \left\{ d(X, \tau) \frac{\partial \tau}{\partial t} + b(X, \tau) u_k \frac{\partial \tau}{\partial x_k} \right\} = \frac{\partial}{\partial x_k} \left(x_2^\sigma a(X, \tau) \frac{\partial \tau}{\partial x_k} \right) + x_2^\sigma F, \quad X \in \Omega, \tag{3}$$

where

$$d(X, \tau) = \rho_j(X) c_j(X, \tau), \quad X \in \Omega_j, \; j = 1, 2, 3; \qquad b(X, \tau) = \begin{cases} 0, & X \in \Omega_1 \cup \Omega_3 \\ \rho_2 C_2(r), & X \in \Omega_2 \end{cases};$$

$$a(X, \tau) = \lambda_j(X, \tau), \quad X \in \Omega_j, \; j = 1, 2, 3;$$

$$F \equiv \begin{cases} \rho_j(X) e_j(X, t), \quad X \in \Omega_j, \; j = 1, 3; \\ \rho_2 e_2(X, t) + \rho_2 f_k(X, t, \tau) u_k + \nu(\tau) \left\{ 2 \left(\frac{\partial u_k}{\partial x_k} \right)^2 + 2\sigma \left(\frac{u_2}{x_2} \right)^2 + \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right)^2 \right\}, \quad X \in \Omega_2; \end{cases} \quad \rho_2(X) \equiv \rho_2; \; \lambda_2(X, \tau) \equiv \lambda_2(\tau);$$

$$c_2(X, \tau) \equiv c_2(\tau).$$

In Eqs. (1), (2) and (3) the summation is carried out over the recurrent index k; $\sigma = 0$ in a plane channel and $\sigma = 1$ in an axisymmetric channel. The conjugation conditions (4) and (5) and the initial conditions (6) are set up at the liquid–wall contact surfaces:

$$[\tau]|_{\Gamma_k} = 0, \quad k = 1, 2; \tag{4}$$

$$\left[\lambda(X,\ \tau)\frac{\partial\tau}{\partial x_2}\right]\Big|_{\Gamma_k}=Q_k(X,\ t,\ \tau)\equiv(-1)^{k-1}\rho_{wk}u_{wk}(h_{wk}(\tau)-h^0_{wk}+r_k),\quad k=1,\ 2; \tag{5}$$

$$\bar{u}(X,\ 0)=\bar{u}_0(X),\quad q(X,\ 0)=q_0(X),\quad X\in\Omega_2;\quad \tau(X,\ 0)=\tau_0(X),\quad X\in\Omega. \tag{6}$$

The assumptions made concerning the character of the flow and heat transfer near the channel exit make it possible to write the boundary conditions in the form

$$\bar{u}=\bar{u}_s(X,\ t),\quad X\in\partial\Omega_2\setminus S_4'; \tag{7}$$

$$u_2=0,\quad \frac{\partial u_1}{\partial x_1}=0,\quad X\in S_4'; \tag{8}$$

$$q(X,\ t)=q_s(t),\quad X\equiv(x_1,\ x_2):\ (x_1-\delta,\ x_2)\in S_4',\quad \forall\delta>0; \tag{9}$$

$$\tau=\tau_s(X,\ t),\quad X\in\partial\Omega\setminus S_4'; \tag{10}$$

$$\lambda(\tau)\frac{\partial\tau}{\partial x_1}=Q(t),\quad X\in S_4'. \tag{11}$$

Because of the temperature dependence of the viscosity ν and the mass force $\bar{f}$, the liquid flow cannot be calculated independently of the heat transfer, and Eqs. (1)-(3) must be solved jointly. We make the following assumptions: The temperature τ does not go beyond the limits of some positive range; the liquid velocity, the spatial derivatives of the temperature and velocity, and the quantity $|u_2x_2^{-\sigma}|$ are bounded by a constant $D>0$; the temperature-dependent coefficients of the problem (1)-(11) are bounded in any positive range of temperature variation. These assumptions make it possible, as in [2, 3], to reduce the problem (1)-(11) to a problem with limited nonlinearity. To shorten the computations, we assume that these transformations have been performed and the values of the functions ν, c_j, λ_j, $j=1,\ 2,\ 3$, do not go beyond the limits of a positive range, while the functions Q_i, f_i, $i=1, 2$, are bounded. The problem obtained is approximated by an economical nonlinear difference scheme of fractional steps. For this we introduce a grid in the calculated region which is nonuniform in time and in the transverse coordinate and uniform lengthwise and which includes fractional and whole steps in time. Henceforth an index h to the designation of a function, a region, or its boundary means that the reduction of the function to the grid or to the grid analog of the region (boundary) is being considered. We approximate Eqs. (1) and (2) by a difference scheme, implicit with respect to velocity and explicit with respect to pressure, which coincides with those in [3, 4] for constant thermophysical properties of the liquid:

$$\varepsilon\frac{R^{m+1}-R^m}{\Delta t_{m+1}}+(x_2^\sigma v_i^{m+\frac{1}{2}})_{\bar{x}_i};\quad X\in\Omega_{2h}\cup\Gamma_{1h}\cup S_{4h}'; \tag{12}$$

$$\frac{x_2^\sigma}{\Delta t_{m+1}}(v_i^{k+\frac{1}{2}}-v_i^k)+L_i^k(v_i^{k+\frac{1}{2}},\ \alpha T^{k+\frac{1}{2}})=\delta_k^m\Big\{-x_2^\sigma R^m_{x_i}-$$
$$-\sigma\delta_i^2\,2\nu(\alpha T'^{m+\frac{1}{2}})\left(\frac{\hat{v}_2}{x_2}\right)^{m+\frac{1}{2}}\Big\}+\frac{1}{2}f_{ih}(X,t^{k+\frac{1}{4}},\ \alpha T^{k+\frac{1}{2}}),\ X\in\Omega_{2h}, \tag{13}$$

where

$$L_i^k(\omega^q,\ \alpha T^{k+\frac{1}{2}})\equiv\frac{1}{2}\{x_2^\sigma\bar{v}_j^{-j\,k}\zeta(\bar{v}_j^{-j\,k})\omega^q_{\bar{x}_j}+x_2^\sigma v_j^k\zeta(v_j^k)\omega^q_{x_j}\}-$$
$$-(x_2^\sigma\nu(\alpha T^{k+\frac{1}{2}})\omega^q_{x_j})_{\bar{x}_j}-\delta_k^{m+\frac{1}{2}}\Big\{\frac{1}{2}(x_2^\sigma\nu(\alpha T^{m+1})v^{m+1}_{j\bar{x}_i})_{\bar{x}_j}+$$
$$+\frac{1}{2}(x_2^\sigma\nu(\alpha T^{m+1})v^{m+1}_{j\bar{x}_i})_{x_j}+(x_2^\sigma\nu(\alpha T^{m+1})v^{m+1}_{ix_i})_{\bar{x}_i}\Big\}. \tag{14}$$

We approximate Eq. (3) by a scheme of fractional steps:

$$x_2^\sigma d_h(X,\ \alpha T^{k+\frac{1}{2}}) \frac{T^{k+\frac{1}{2}} - T^k}{\Delta t_{m+1}} = (x_2^\sigma a_h(X,\ \alpha T^{k+\frac{1}{2}}) \times$$
$$\times \zeta(T_{x_p}^{k+\frac{1}{2}})\, T_{x_p}^{k+\frac{1}{2}})_{\bar{x}_p} - x_2^\sigma b_h(X,\ \alpha T^{k+\frac{1}{2}})\, v_p^{k+\frac{1}{2}} \zeta(v_p^{k+\frac{1}{2}})\, T_{x_p}^{k+\frac{1}{2}} - \quad (15)$$
$$-\delta_k^{m+\frac{1}{2}} \left\{ \tilde{F}_h x_2^\sigma - \delta(\Gamma_{kh}) \frac{Q_k(X,\ t^{m+\frac{1}{4}},\ \alpha T^{m+1})}{H2_{Nk}} \right\},\quad X \in \Omega_h.$$

The initial conditions (6) and the boundary conditions (7) and (10) are approximated by a simple reduction to the grid if the functions assigning them are continuous. Otherwise, Steklov averagings over cells of the grid are used. The grid boundary conditions (16) for the longitudinal velocity at a fractional layer are obtained by writing (13) for S'_{4h} with allowance for (8), as in [6]. At a whole layer we approximate (8). The boundary conditions (18) and (19) for the temperature at S'_{4h} are obtained by approximating (11):

$$\frac{x_2^\sigma}{\Delta t_{m+1}} (v_1^{m+\frac{1}{2}} - v_1^m) + x_2^\sigma v_1^m v_{1\bar{x}_1}^{m+\frac{1}{2}} + \frac{2x_2^\sigma}{H} \nu(\alpha T^{m+\frac{1}{2}})\, v_{1\bar{x}_1}^{m+\frac{1}{2}} + \quad (16)$$
$$+ x_2^\sigma \frac{R^m - q_s(t^m)}{H} = \frac{1}{2} f_{1h}(X,\ t^{m+\frac{1}{4}},\ \alpha T^{m+\frac{1}{2}}),\quad X \in S'_{4h};$$

$$v_1^{m+1} = \overset{-1}{v_1^{m+1}},\quad X \in S'_{4h}; \quad (17)$$

$$x_2^\sigma \rho_2 c_{2h}(\alpha T^{m+\frac{1}{2}}) \frac{T^{m+\frac{1}{2}} - T^m}{\Delta t_{m+1}} = -\frac{x_2^\sigma}{H} \lambda_{2h}(\alpha \overset{-1}{T}{}^{m+\frac{1}{2}})\, \zeta(T_{\bar{x}_1}^{m+\frac{1}{2}})\, T_{\bar{x}_1}^{m+\frac{1}{2}} + Q(t^{m+\frac{1}{2}}); \quad (18)$$

$$\lambda_{2h}(\alpha T^{m+1})\, T_{\bar{x}_1}^{m+1} = Q(t^{m+1}),\quad X \in S'_{4h}. \quad (19)$$

In Eqs. (12)-(19) $i = 1, 2$; $k = m, m + 1/2$; $m = 0, 1, 2, \ldots, M - 1$;

$$\delta_l^p = \begin{cases} 1, & l = P \\ 0, & l \neq P \end{cases};\quad \delta(\Gamma_{kh}) = \begin{cases} 1, & X \in \Gamma_{kh} \\ 0, & X \notin \Gamma_{kh} \end{cases};\quad j = \begin{cases} 2, & i=1,\ k=m+\frac{1}{2} \\ 1, & i=2,\ k=m+\frac{1}{2} \\ i, & k = m \end{cases};$$

$$P = \begin{cases} 1, & k = m \\ 2, & k = m + \frac{1}{2} \end{cases};\quad \alpha T^{k+\frac{1}{2}} \equiv \alpha T^{k+\frac{1}{2}} + (1-\alpha)\, T^k,\ \alpha \in [0,1];$$

$$\tilde{F} = \begin{cases} \rho_i(X)\, e_i(X,\ t^{m+\frac{3}{4}}), & i = 1, 3,\ X \in \Omega_{ih}; \\ \nu(\alpha T^{m+1})\, \rho_2 \hat{\Phi} + \rho_2 \left\{ \sum_{i=1}^{2} f_{ih}\, v_i^{m+1} \zeta(v_i^{m+1}) + e_2(X,\ t^{m+\frac{3}{4}}) \right\}, & X \in \Omega_{2h}; \end{cases}$$

$$\hat{\Phi} = \left\{ 2\left[(v_{1x_1}^{m+1})^2 + (v_{2x_2}^{m+1})^2 + \sigma\left(\frac{v_2^{m+\frac{1}{2}}}{x_2}\right)^2 \right] + (v_{1x_2}^{m+1} + v_{2x_1}^{m+1})^2 \right\} \zeta(v_{ix_j}^{m+1})\, \zeta\left(\frac{v_2^{m+1}}{x_2^\sigma}\right);$$

$$v_{i\,\tilde{x}_2}^{m+1} = \begin{cases} v_{ix_2}^{m+1}, & X \in \bar{\Omega}_{2h} \setminus \Gamma_{1h}; \\ v_{i\bar{x}_2}^{m+1}, & X \in \Gamma_{1h}, \end{cases}$$

$\tilde{\varphi}$ is the piecewise-constant fill-in of the grid function φ; φ_{x_i} and $\varphi_{\bar{x}_i}$ are the right and left difference quotients of φ with respect to the coordinate x_i; $\overset{-1}{\varphi}$ is a shift of the function one step to the left along the coordinate x_i; $\zeta(a)$ is a smooth function, monotonic with respect to $|a|$, equal to unity at $|a| < D$ and to zero at $|a| \geq 2D$ [2-5].

For the difference scheme (12)-(19) constructed, on the assumption of the existence of a general solution $\bar{u}$, q, τ of the problem having a certain smoothness, by the methods of [2-5] one can prove the convergence of an approximate solution $\bar{v}$, R, T to the exact solution and obtain an estimate of the rate of convergence in an energy norm.

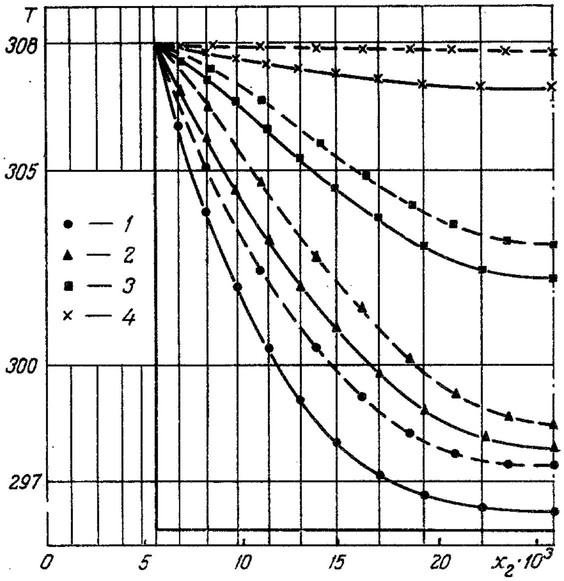

Fig. 2. Profiles of temperature T, °K, in an established regime as a function of x_2, m, in different channel cross sections: solid curves) data of calculation; dashed curves) data of [7]; 1) x_1 = 0.0774 m; 2) 0.296; 3) 0.795; 4) 3.375.

The nonlinear system of algebriac equations (12)-(19) is solved by the iteration method at each step in time. In the process, the parameters of the preceding iteration are substituted into the coefficients of the equations, after which each equation is solved by the Monte-Carlo method. We note that, as in [4], the scheme (12)-(19) does not require the assignment of boundary conditions at the axis of a cylindrical channel.

The method presented was realized in the form an an ALGOL program for a BESM-6 computer. The program was checked out on the example of a calculation of the steady heat transfer exchange under conditions of mixed convection during air flow in a vertical copper pipe with isothermal walls, which was studied numerically and experimentally in [7]. The problem of achieving agreement of the results was set up by calculating the process by the relaxation method. The Targ solution for the hydrodynamic initial section of a cylindrical channel was assigned as the hydrodynamic initial condition. The results of the calculations for the case of isothermal flow practically coincided with the results of [7]. The results of the calculation of mixed convection are presented in Fig. 2. The disagreement between the results and those in [7] is evidently explained by the fact that the smallness of the characteristic temperature drop leads to slow development of the process in a calculation by the relaxation method. This prevented an approximation to the established state. Nevertheless, the uniform distribution of the wall temperature confirms the assumption made in [7] that the walls of the pipe of the experimental installation used are thermally thin. The compiled program made it possible without the enlistment of external devices to take 20 nodes along the length of the channel and 30 (20 in the liquid and 10 in the wall) along the cross section.

NOTATION

u_1 is the longitudinal velocity;
u_2 is the transverse velocity;
τ is the temperature;
q is the pressure;
ν is the kinematic viscosity;
ρ is the density;
C is the heat capacity;
λ is the thermal conductivity;
e is the specific power of heat release;
$\vec{f}$ is the mass force;
h is the enthalpy;

r is the latent of vaporization;
t is the time;
X is the point of the plane;
x_1 is the longitudinal coordinate;
x_2 is the transverse coordinate;
Δt is the step in time;
H is the step along x_1;
H2 is the step along x_2;
ε is the parameter of difference scheme.

Indices

0 is the initial;
s is the boundary;
m is the number for time point;
w is the number for coolant parameters;
h is the number for grid function.

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METHOD OF EXTENSION OF THE DOMAIN OF HEAT- AND MOISTURE-CONDUCTION PROBLEMS

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UDC 536.24.02

A method based on extension of the domain of the problem is applied to the solution of parabolic differential equations in heat- and moisture-conduction problems.

There is a well-known method for the solution of elasticity problems by extension of the domain of definition [1, 2]. A similar approach is possible in heat- and moisture-conduction problems for the solution of differential equations of parabolic type.

Let it be required to determine a function $T(r, \tau)$ continuous and defined in a closed domain D, in which it satisfies the differential equation

$$\frac{\partial T(r, \tau)}{\partial \tau} = a\nabla^2 T(r, \tau) + \varphi(r, \tau), \tag{1}$$

the initial condition

$$v(r) = T(r, 0), \tag{2}$$

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